

## Statistics 433: Stochastic Processes

### General remarks:

- \* If you haven't registered please let me know via e-mail.
- \* Drop HW in box.
- \* Class TTh 1:30-3:00, in F38 JMHH
- \* The target goals are:

Description: This course is to be a basic introduction to stochastic processes. The primary focus will be on Markov chains both in discrete time and in continuous time. By focusing attention on Markov chain, we can discuss many interesting models (from physics to economics). Topics covered include: stable distributions, birth-death processes, Poisson processes, time reversibility, random walks, Brownian motion and Black-Scholes.

Prerequisites: Stat 430 or permission of instructor.

Instructor: Lawrence Shepp.

Book: Introduction to Stochastic modeling, by Taylor and Karlin, 3<sup>rd</sup> edition.

Workload: This course will be driven primarily by problems. Hence there will be problem sets due weekly.

Homework = 40% (possibly including some computing / simulations)

Midterm = 30%

Final = 30%

<http://stat.wharton.upenn.edu/~shepp/teaching/433>

(or follow links from Shepp's home page)

### Time allocation:

Chapters 1 & 2: Review of probability (1 weeks)

Chapter 3: Markov chains (5 weeks)

Chapter 4: Long run behavior of Markov chains (2 weeks)

Chapter 5: Poisson processes (3 weeks)

Chapter 6: Continuous time Markov chains (3 weeks)

Chapter 8: Brownian motion (as time permits)

Details of topics to be covered:

1. A brief review of probability
1. What is probability?
2. Rules of probability
3. Conditional probability
4. Independence
5. Some urn models and useful probability models
6. Random variables, random vector and random processes
7. Some useful random variables and processes
8. Expectations
9. Joint distributions
10. Moment generating, characteristic, and probability generating
11. Applications and examples

## 2. A brief review of some useful inequalities and limiting theorems

1. Markov's inequality
2. Chebyshev's inequality
3. Law of large numbers
4. Central limit theorem
5. Applications and examples

## 3. Conditional probability and conditional expectation

1. Discrete case
2. Continuous case
3. Computing expectations by conditioning
4. Computing probabilities by conditioning
5. Applications and examples

## 4. Markov chains

1. Definition
2. Initial distribution and transition probability
3. Markov chains having two states
4. Applications and examples
5. Computations with transition probabilities
6. Hitting times
7. Chapman-Kolmogorov equations
8. Classification of states
9. Induced martingales
10. Birth and death chains
11. Limiting and stationary probabilities of Markov chains
12. More applications and examples
13. Some related topics

## 5. Exponential distribution and Poisson process

1. Properties of exponential distribution
2. Counting process
3. Poisson process
4. Inter-arrival and waiting time distributions
5. Further properties of Poisson process
6. Related topics (non-homogeneous, compound processes,...)
7. Applications and examples

## 6. Continuous-time Markov chains

1. Definition
2. Birth and death process
3. The Kolmogorov differential equations
4. Limiting probabilities
5. Time reversibility
6. Computing the transition probabilities

## 7. Applications and examples

### 7. Brownian motion

1. Random walks and Brownian motion
2. Geometric Brownian Motion
3. Black-Scholes Option Pricing Formula

\* Stat lab F96 JMHH- M-F, 3-6 PM

\* Office hours: Wednesday 2:00-3:30, Huntsman 462, or via e-mail.

\* TA: Pengyuan Wang, Office hours: Tues, Thurs 10:30-11:30 AM (HW due by Friday noon).

\* Book: Intro to Stochastic Modeling, by Taylor and Karlin, 3rd ed.

\* Workload: Course is problem driven. Problem sets will be due weekly.

\* HW = 40%

\* Midterm = 30%

\* Final = 30%

Homework is turned in at the start of class

### Remarks:

1. I will miss the first 3 classes; however there will be able replacements for me.

2. I am new to U Penn - I spent 34 years at Bell Labs and 13 years at Rutgers.

That I am not so young is good for you; I have a lot of experience.

3. I am a firm believer that the theory of probability lies at the center of science and will do my best to convince you of this.

4. I object to the use of a syllabus but will conform to the request as I work for U Penn and (in a way) for you, the student. However, please do not expect me to follow it when it makes no sense to do so.

5. I will use my web page to post notes, homework solutions, results of quizzes etc. Please monitor it: [stat.wharton.upenn.edu/~shepp](http://stat.wharton.upenn.edu/~shepp). Also please monitor this syllabus page (maybe).

### Approximate timemarks (dates subject to change):

\* Sep 9: Chapters 1 & 2: Review of probability (Read: pp 1-15, 57 - 68).

Start doing HW1.

\* Sep 14: Section 2.5: Martingales (Questions ok on HW1 Read: pp 60-61, and 87 - 93)

\* Sep 16: Markov chains 1 (p 95 - 102)

\* Sep 17 @ noon: HW 1 due

\* Sep 21: Markov chains 2 (p 105-112)

\* Homework 1 solutions will have been posted...

\* Sep 23: Markov: (p 116 - 124)

\* Sep 24 @ noon: Homework 2 due

\* Sep 28: Markov: examples (p 135 - 147)

\* Sep 30: Markov: Random walks (141 - 166)

\* Oct 1 @ noon: Homework 3 due

\* Oct 5: Markov: Random walks

\* Oct 7: Branching processes (III.8)

\* Oct 8: Homework 4 due

- \* Oct 12: Branching and generating functions (III.9)
- \* Oct 14: Review
- \* Oct 15: (III.7) Homework 5 due
- \* Oct 19: Midterm
- \* Oct 21: Chapter IV: Long run behavior of Markov chains
- \* Oct 22: Homework 6 due. Answers posted to Midterm.
- \* Oct 26: IV.2: Models
- \* Oct 28: (IV.3)
- \* Oct 29: Homework 7 due
- \* Nov 2: V.1: The Poisson distribution (V.1 and V.2)
- \* Nov 4: V.2: Rare events (V.3 and V.4 up to page 305)
- \* Nov 5: Homework 8 due
- \* Nov 9: V.3: Gamma
- \* Nov 11: Section V.3
- \* Nov 12: Homework 9 due
- \* Nov 16: V.4: connection to uniform distribution (read VI.4 and VI.5)
- \* Nov 18: Chapter 6: Continuous time Markov chains
- \* Nov 19: Homework 10 due
- \* Nov 23: Chapter 6.2, 6.3: Birth / Death processes (finish reading VI.1-5)
- \* Thanksgiving break
- \* Nov 30: Chapter 6.3 and 6.4
- \* Dec 2: Chapter 8: Brownian motion 8.2: Reflection
- \* Dec 3: Homework 11
- \* Dec 7: 8.2 8.3
- \* Dec 9: 8.4: Models; Drift and stock prices
- \* Dec 10: Homework 12 (LAST DAY OF CLASS)
- \* Dec 14: 2pm - 4pm. Review session (by ?)
- \* Dec ? : ?-?: Final exam, ? JMHH.