BEPP 931
NUMERICAL METHODS IN ECONOMICS
Location: SHDH ???
Meeting Time: Fall 2017, Fri. 9-noon
Professor Ulrich Doraszelski

CONTACT INFORMATION

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COURSE DESCRIPTION

The objective of this course is to introduce graduate students to computational approaches for solving economic models. We will formulate economic problems in computationally tractable form and use techniques from numerical analysis to solve them. Examples of computational techniques in the current economics literature as well as areas where these techniques may be useful in future research will be discussed. We will pay particular attention to methods for solving dynamic optimization problems and computing equilibria of games. The substantive applications will cover a wide range of problems including industrial organization, game theory, macroeconomics, finance, and econometrics.

READINGS

You are expected to have read the assigned book chapters and papers before coming to class.

Textbook:
- Judd, K. (1998) "Numerical Methods in Economics" MIT Press

Books on Reserve (Lippincott Library Reserve in Van Pelt):
- Chiang, A. (1992) "Elements of Dynamic Optimization" McGraw-Hill
- Zangwill, W. & Garcia, C. (1981) "Pathways to Solutions, Fixed Points, and Equilibria" Prentice-Hall
- Miranda, M. & Fackler, P. (2002) "Applied Computational Economics and Finance" MIT Press
- Stokey, N. & Lucas, R. (1989) "Recursive Methods in Economics Dynamics" Harvard University Press

Papers:
Papers can be downloaded freely from the Penn Libraries.

Course Webpage:
Lecture notes, problem sets and solutions as well as additional materials are posted on Canvas at ???.

PROBLEM SETS AND TERM PAPER

Your course grade will be based on the problem sets (50%) and on the term paper (50%).

Problem Sets and In-Class Exercises:
Doing computation is the only way to learn computation. The problem sets are designed to get you to apply various numerical methods. You are welcome to work in groups of two or three. More generally, I strongly encourage you to discuss the problem sets with your classmates. Classmates are also a valuable source for tips on programming and general computer advice.

A write-up of your solution (one write-up per group) is due in class on the date indicated on the problem set. The write-up should

- provide a brief and clear verbal description of the methods employed;
- use tables and/or graphs to describe your results;
- contain your computer programs in an appendix.

Just handing in computer programs and their output is not acceptable. Please remember to indicate the names of all group members on the write-up.

Some of the exercises will be discussed in class. These exercises are clearly labeled on the problem set and in the class schedule. Prior to each session work through the assigned readings and do the in-class exercises. Depending on the session I may assign different in-class exercises to different students. During class I will call on one or more students to present their solution to the in-class exercises. You will explain your solution to your classmates and illustrate it on the classroom computer. This means that you will have to bring your computer programs to class on a memory stick or your laptop.

Note that the goal is not to solve each and every exercise; instead the problem sets are meant to give you an opportunity to practice solving economic models on a computer.

Term Paper:
You have considerable freedom to follow your interests in choosing a topic for a term paper. The paper should

- motivate and formulate a research question;
- explain why numerical methods are needed to answer the question;
- present a model;
- discuss a computational strategy for solving the model;
- present the results;
- evaluate the accuracy of the results.

The term paper should be at most 15 double-spaced pages excluding tables, graphs, and appendices (12 point font, 1 inch margins).

I urge you to start thinking of a topic early in the semester. The term paper could involve replicating a computational paper. Another way to develop a topic is to think of a theoretical paper that does not compute any (except perhaps trivial) examples and to compute a small set of interesting examples. Ideally, however, the term paper would ask (and answer) an original research question. Once you have identified a topic, please write a short (at most one page) proposal. Email the proposal to me and make an appointment to discuss the suitability of the topic. I expect all students to have discussed a topic with me before Fri., October 20, 2017.

The term paper itself should formally define the problem and emphasize the numerical methods used to solve it (pros and cons of the chosen approach, available alternative approaches). It should further include an error analysis (compute and verify).

You will give a short presentation of your term paper in one of the last two sessions. The goal is to help you organize your thoughts and obtain feedback from your classmates.

The final version of the paper is due Fri., December 15, 2017.

SOFTWARE AND HARDWARE

There is no point in re-inventing the wheel, and I encourage you to use canned code whenever possible. The web is a valuable resource of ready-made code which is often written by professional programmers and/or applied mathematicians and has been debugged and optimized over many years. However, part of this course is to acquaint you with different numerical methods and programming a method is the best way to understanding how it works. Using canned code is not. I ask you to rely on your own judgment in making this decision. As a rule of thumb I suggest that in a session on optimization, say, you do not use an existing optimization routine but instead code your own.

Matlab:
The default computer language for this course is Matlab. The complete documentation of Matlab and its toolboxes can be freely downloaded at www.mathworks.com.
A tutorial to get you started and programming tips are available at
www.mathworks.com/access/helpdesk/help/pdf_doc/matlab/getstart.pdf
www.mathworks.com/access/helpdesk/help/pdf_doc/matlab/programming_tips.pdf
Wharton has a site license for Matlab that covers Wharton-owned computers. For access please contact Kenny Davis.

Mario Miranda and Paul Fackler have written a Matlab toolbox called CompEcon. The toolbox includes equation solvers and optimization routines, a set of routines for function approximation, a set of numerical integration routines, routines for solving ordinary differential equations (both initial and boundary value problems), and routines for solving discrete- and continuous-time dynamic programming problems. The toolbox can be freely downloaded at www4.ncsu.edu/~pfackler/compecon/toolbox.html. Its use is explained in detail in Miranda & Fackler's textbook (on reserve).

Wharton HPC cluster:
Wharton has a high-performance linux-based cluster that you can use for the purposes of this course and your own research. In encourage you to read up on it at http://research-it.wharton.upenn.edu/documentation/ and to request an account. Amongst other software packages the grid has Matlab. The CompEcon toolbox is installable by any user by running /usr/local/demo/Matlab/setup_compecon.sh.

CLASS SCHEDULE
(subject to change)

SESSION 1: SEPTEMBER 1, 2017

Part 1: Introduction

Introduction to Numerical Methods

Topics:

- Course mechanics
- Roles of computation in economic analysis:
  - Generate hypothesis and counterexamples
  - Assess quantitative significance
  - Relax "technical" assumptions
  - Analyze complex models
- Deductive vs. computational theory
- A toolkit

Readings:

- Judd, Chap. 1
- Judd, K. (1997) "Computational Economics and Economic Theory: Substitutes or Complements?" *Journal of Economic Dynamics and Control* 21 (6) pp. 907-942
- Kubler, F. (2007) "Approximate Generalizations and Computational Experiments" *Econometrica* 75 (4) pp. 967-992 (optional)

Elementary Concepts of Numerical Analysis

Topics:

- Computer arithmetic
- Error analysis:
  - Sources of error
  - Error propagation
  - Controlling the error
  - Compute and verify
- Algorithmic complexity: speed and memory

Readings:

- Judd, Chap. 2

To-dos:

- Come to class prepared to name at least one paper that has (or should have) used numerical methods. What is (or would be) the value added by numerical methods?

SESSION 2: SEPTEMBER 8, 2017

Part 2: Basic Concepts

Linear Equations

Topics:
- Direct methods: Backsubstitution, LU and other decompositions
- Error analysis and condition numbers
- Iterative methods: Gauss-Jacobi and Gauss-Seidel
- Acceleration and stabilization methods

Readings:
- Judd, Chap. 3

Optimization

Topics:
- Comparison methods
- Newton's method
- Direction set methods
- Constrained optimization

Readings:
- Judd, Chap. 4, Sections 4.1-4.7

To-dos:
- Problem Set 1 due

SESSION 3: SEPTEMBER 15, 2017

Optimization

Topics:
- Applications:
  - Portfolio choice (Exercise 3-1)
  - Life-cycle consumption (Exercise 3-2)
  - Principal-agent and adverse selection problems (Exercise 3-3)
  - Normal form games with discrete actions (Exercise 3-4)
- Practical considerations:
  - Nature of problem and choice of algorithm
  - Local and global optima
  - Hot starts and cold starts
  - Canned code

Readings:
- Judd, Chap. 4, Sections 4.8-4.14

- McCullough, B. & Vinod, H. (2003) "Verifying the Solution from a Nonlinear Solver: A Case Study" *The American Economic Review* 93 (3) pp. 873-892 (optional)
- Shachar, R. & Nalebuff, B. (2004) "Verifying the Solution from a Nonlinear Solver: A Case Study: Comment" *The American Economic Review* 94 (1) pp. 382-390 (optional)
- McCullough, B. & Vinod, H. (2004) "Verifying the Solution from a Nonlinear Solver: A Case Study: Reply" *The American Economic Review* 94 (1) pp. 391-396 (optional)
- Drukker, D. & Wiggins, V. (2004) "Verifying the Solution from a Nonlinear Solver: A Case Study: Comment" *The American Economic Review* 94 (1) pp. 397-399 (optional)

Nonlinear Equations

Topics:
- Bisection method
- Gauss-Jacobi and Gauss-Seidel methods
- Fixed-point iteration
- Newton's method
- Powell's hybrid method

Readings:
- Judd, Chap. 5, Sections 5.1-5.7

To-dos:
- In-class discussion of Exercises 3-1, 3-2, 3-3, and 3-4 from Problem Set 3
- Problem Set 2 due

SESSION 4: SEPTEMBER 22, 2017

Nonlinear Equations

Topics:
- Applications:
    - Competitive general equilibrium (Exercise 4-1)
    - Asset pricing (Exercise 4-2)
    - Normal form games with continuous actions (Exercise 4-3)
- Continuation and homotopy methods

Readings:
- Judd, Chap. 5, Sections 5.8-5.12
- Zangwill, W. & Garcia, C. (1981), Chap. 1, 2 (less technical, optional)
- Allgower, E. & Georg, K. (1993) "Continuation and Path Following" *Acta Numerica* 2 pp. 1-64 (more technical, optional)

Function Approximation

Topics:

- Approximation vs. statistics
- Local approximation methods:
  - Taylor series expansion
  - Padé expansion
- Global approximation methods:
  - L^p approximation: orthogonal polynomials
  - Interpolation methods: Lagrange, Hermite, and piecewise polynomial (spline) interpolation
  - Regression methods: Chebyshev regression

Readings:

- Judd, Chap. 6, Sections 6.1-6.10

To-dos:

- In-class discussion of Exercises 4-1, 4-2, and 4-3 from Problem Set 4
- Problem Set 3 due

SESSION 5: SEPTEMBER 29, 2017

Function (and Set) Approximation

Topics:

- Shape-preserving methods
- Multidimensional methods:
  - Tensor products and complete polynomials
  - Chebyshev regression
  - Finite element methods
  - Neural networks
- Set approximation and application to repeated games

Readings:

- Judd, Chap. 6, Sections 6.11-6.15
- Judd, K., Yeltekin, S. & Conklin, J. (2003) "Computing Supergame Equilibria" *Econometrica* 71 (4) pp. 1239-1254
- Abreu, D., Pearce, D. & Stacchetti, E. (1990) "Toward a Theory of Discounted Repeated Games with Imperfect Monitoring" *Econometrica* 58 (5) pp. 1041-1063 (optional)

Numerical Integration and Differentiation

Topics:

- Newton-Cotes formulas

- Gaussian formulas
- Multidimensional integration: product rules and nonproduct rules
- Numerical differentiation

Readings:
- Judd, Chap. 7

To-dos:
- Problem Set 4 due

SESSION 6: OCTOBER 13, 2017

Monte Carlo and Quasi-Monte Carlo Methods

Topics:
- Pseudorandom number generation
- Monte Carlo integration
- Quasi-Monte Carlo integration
- Stochastic approximation and stochastic search
- Application: Portfolio choice

Readings:
- Judd, Chap. 8, 9

Part 3: Methods for Functional Equations

Finite-Difference Methods

Topics:
- Initial value problems:
    - Euler methods
    - Runge-Kutta methods
- Boundary value problems: shooting
- Application to finite- and infinite-horizon optimal control problems
- Integral equations

Readings:
- Judd, Chap. 10
- Chiang, A. (1992), Chap. 7, Sections 7.1-7.4, Chap. 8, Section 8.2, Chap. 9, Section 9.1 (background material on optimal control problems)

To-dos:
- Problem Set 5 due

SESSION 7: OCTOBER 20, 2017

Projection Methods

Topics:
- Choice of basis functions
- Choice of approximation degree
- Choice of projection conditions
- Computation of solution
- Verification of solution

Readings:
- Judd, Chap. 11

Numerical Dynamic Programming

Topics:
- Discrete-time dynamic programming
- Continuous-time dynamic programming
- Methods for finite-state problems:
  - Value function iteration
  - Policy function iteration
  - Gaussian acceleration methods
- Methods for continuous-state problems:
  - Discretization
  - Parametric approximation methods
  - Projection methods

Readings:
- Judd, Chap. 12
- Lecture notes "Review of Dynamic Programming"
- Stokey, N. & Lucas, R. (1989), Chap. 3, 4, 9 (background material on dynamic programming)

To-dos:
- Problem Set 6 due

SESSION 8: OCTOBER 27, 2017

Introduction to the Wharton HPC Cluster

Guest speaker: Hugh MacMullan, Wharton IT

SESSION 9: NOVEMBER 3, 2017

Projection Methods

Topics:

- Applications:
  - Job market signaling (Exercise 8-1)
  - Learning-by-doing (Exercise 8-2)
  - Life-cycle consumption (Exercise 8-3)
  - Optimal growth (Exercise 8-4)

Readings:

- Spence, M. (1974) "Competitive and Optimal Response to Signals: An Analysis of Efficiency and Distribution" *Journal of Economic Theory* 7 (3) pp. 296-332. (optional)

Perfect Foresight Models

Topics:

- Time domain methods:
  - Tatonnement method
  - Fixed-point iteration
  - Newton's method
  - Parametric path method
  - Fair-Taylor method
- Recursive methods:
  - Projection methods
  - Time iteration
  - Fixed-point iteration
- Applications:
  - OLG (Exercise 8-5)
  - Optimal growth (Exercise 8-6)
  - Time-consistent taxation (Exercise 8-7)

Readings:

- Judd, Chap. 16

To-dos:

- In-class discussion of Exercises 8-1, 8-2, 8-3, 8-4, 8-5, 8-6, and 8-7 from Problem Set 8
- Problem Set 7 due

SESSION 10: NOVEMBER 10, 2017

Rational Expectations Models

Topics:

- Recursive methods:
  - Simulation method
  - Integral equation method

    - Projection methods
    - Time iteration
    - Fixed-point iteration
    - Wright-Williams smoothing
- Applications:
    - Asset pricing (Exercise 9-1)
    - Commodity storage (Exercises 9-2 and 9-3)

Readings:
- Judd, Chap. 17

Dynamic Games with Finite States

Topics:
- From dynamic programming to dynamic games
- Value function iteration approach
- Markov-perfect industry dynamics
- Existence, purification, and multiplicity of equilibrium

Readings:
- Doraszelski, U. & Satterthwaite, M. (2010) "Computable Markov-Perfect Industry Dynamics" *Rand Journal of Economics* 41 (2) pp. 215-243
- Pakes, A. & McGuire, P. (1994) "Computing Markov-Perfect Nash Equilibria: Numerical Implications of a Dynamic Differentiated Product Model" *Rand Journal of Economics* 25 (4) pp. 555-589
- Borkovsky, R., Doraszelski, U., & Kryukov, Y. (2012) "A Dynamic Quality Ladder Model with Entry and Exit: Exploring the Equilibrium Correspondence using the Homotopy Method" Quantitative Marketing and Economics 10 (2) pp. 197-229
- Ericson, R. & Pakes, A. (1995) "Markov-Perfect Industry Dynamics: A Framework for Empirical Work" *Review of Economic Studies* 62 pp. 53-92 (optional)

To-dos:
- In-class discussion of Exercises 9-1, 9-2, and 9-3 from Problem Set 9
- Problem Set 8 due

SESSION 11: NOVEMBER 17, 2017

Dynamic Games with Finite States

Topics:
- Applications:
    - Quality ladder model (Exercise 10-3)
    - Learning-by-doing model (Exercise 10-4)
- Computing all equilibria: Homotopy method

- Computational burden
- Open questions

Readings:

- Besanko, D., Doraszelski, U., Kryukov, Y. & Satterthwaite, M. (2010) "Learning-by-Doing, Organizational Forgetting, and Industry Dynamics" *Econometrica* 78 (2) pp. 453-508
- Borkovsky, R., Doraszelski, U., & Kryukov, Y. (2010) "A User's Guide to Solving Dynamic Stochastic Games Using the Homotopy Method" *Operations Research* 58 (4) pp. 1116-1132 (optional)
- Cabral, L. & Riordan, M. (1994) "The Learning Curve, Market Dominance, and Predatory Pricing" *Econometrica* 62 (5) pp. 1115-1140 (optional)

Dynamic Games with Continuous States

Topics:

- Discrete-time dynamic games with continuous states
- Continuous-time dynamic games with continuous states
- Closed-form solutions:
  - Linear-quadratic games
  - Linear-state games
  - Exponential games
- Projection methods
- Application: R&D races

Readings:

- Judd, Chap. 12, Section 12.6
- Doraszelski, U. (2003) "An R&D Race with Knowledge Accumulation" *Rand Journal of Economics* 34 (1) pp. 20-42
- Reinganum, J. (1981) "Dynamic Games of Innovation" *Journal of Economic Theory* 25 pp. 21-41 (optional)
- Vedenov, D. & Miranda, M. (2001) "Numerical Solution of Dynamic Oligopoly Games with Capital Investment" *Economic Theory* 18 pp. 237-261 (optional)
- Reynolds, S. (1987) "Capacity Investment, Preemption and Commitment in an Infinite Horizon Model" *International Economic Review* 28 (1) pp. 69-88 (optional)

To-dos:

- In-class discussion of Exercises 10-3 and 10-4 from Problem Set 10
- Problem Set 9 due

SESSION 12: DECEMBER 1, 2017

Wrap-up and Student Presentations

Topics:
- Wrap-up
- Student presentations

To-dos:
- Problem Set 10 due

SESSION 13: DECEMBER 8, 2017

Student Presentations

Topics:
- Student presentations