

**OIDD 653/353 - Mathematical Modeling and Applications in Finance**  
Spring 2021 (*tentative-subject to change*)

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## **INSTRUCTOR(S)**

Professor Gerry Tsoukalas, [gtsouk@wharton.upenn.edu](mailto:gtsouk@wharton.upenn.edu)

## **CLASS HOURS, TEACHING ASSISTANT AND OFFICE HOURS**

Sessions: Lectures are held asynchronous, online. Live sessions/office hours will be available on a weekly basis.

Instructor Online Office Hours: TBD. Location: Zoom

TA Office Hours: Joseph Carlstein, [jc95@wharton.upenn.edu](mailto:jc95@wharton.upenn.edu), TBD, Location: Zoom

## **COURSE OBJECTIVES**

This course focuses on the development of models for the pricing, hedging and management of complex financial instruments and derivative securities. An emphasis is placed on the development of optimization tools and data-driven methods and on their applications to relevant problems faced by financial services firms, in areas such as investment management, quantitative trading and risk management. Extensive cases will be used to illustrate the practical implementation of the modeling tools. Topics include: Pricing and hedging of derivatives (vanilla and exotics), portfolio management/portfolio trading and bond analytics, statistical arbitrage, algorithmic/high-frequency trading and market microstructure. Students will use excel/VBA or Matlab (or other languages if desired) for assigned homeworks.

## **TEXT AND READINGS**

Most of the reading for the course will be in the form of handouts that will be distributed online. Additional (optional) reading will be posted on the course website through canvas. A list of (optional) references is provided at the end.

## **ASSIGNMENTS, QUIZZES AND EXAMS**

There will be a series of take-home homeworks/small projects during the course of the semester, corresponding to the topics covered in the lectures.

## **GRADING**

Grading will be based on take-home assignments (90%), and small quizzes on the lectures (10%). There is no mid-term or final exam.

## **LEARNING TEAMS**

Students will be allowed (but not required) to work **in teams of 2** towards the homeworks. Each student's contribution must be clearly stated on each team submission. Late assignments will not be accepted.

## **ETHICS AND USE OF PRIOR MATERIALS**

**Students are strictly not allowed to use any material from previous years. Sources must be clearly referenced.**

## **SUMMARY OF CLASS SESSIONS (these are estimates depending on the pace of the class)**

### **1. Course Overview & Quantitative Finance in Practice (1 lecture)**

We discuss the course syllabus and give an overview of the financial services industry (differentiating between sell-side and buy-side firms) and the different roles encountered in investment banks, hedge funds and quantitative trading firms, including traders, sales, structurers, strategists, risk managers, financial engineers, quants, auditors, compliance etc.

### **2. Hedging (2-3 lectures)**

We lay out the foundations of hedging which will be used throughout the semester. A hedged portfolio is one that is insulated from market forces. We discuss some basic concepts in hedging and distinguish between model-based and data-based hedging.

*Topics: Scenario-based hedging; Regression hedging; In-sample versus out-of-sample performance.*

### 3. Option Pricing Part I – Option Pricing Theory (3 lectures)

We introduce option pricing theory and the necessary tools to understand it, including: Taylor expansions, stochastic differentiation and integration, Ito's lemma, and martingales. We introduce the law of one-price and no-arbitrage pricing principles, before building intuition on the assumed log-normality of stock prices. We then derive and discuss the Black-Scholes-Merton model for European options.

*Topics: No-arbitrage pricing; Martingales; Risk-neutral measure; Understanding stochastic differential equations; Ito; Geometric Brownian process and the lognormal distribution of stock prices; The Black-Scholes Model;*

### 4. Option Pricing Part II – The Binomial Method (3 lectures)

After a brief introduction on the history of binomial models, we introduce binomial pricing theory and establish the link with the Black-Scholes model. We then apply binomial pricing on simple European calls and puts, before extending it to American options. We continue with binomial pricing methods for exotic options, including: options with path-dependent payoffs and options on multiple underlying assets (three-dimensional binomial trees), and variance swaps. We conclude with a discussion on the limits of binomial pricing in practice. Excel/VBA and Matlab pricing code is developed for all of the examples. Market data is used when appropriate.

*Topics: Binomial Model; European and American options; Path independent and path dependent options: caps, barrier and lookback options; Options on multiple assets.*

### 5. Option Pricing Part III – Monte Carlo Simulation (3 lectures)

After a brief introduction on the history of Monte Carlo simulation and a review of basic statistics, we introduce the theory and apply it to pricing exotic options, including pricing path independent and dependent options. We then discuss popular variance reduction techniques which are used in practice. Excel/VBA and Matlab pricing code is developed for all of the examples. Market data is used when appropriate. We conclude by discussing the limits of Monte Carlo simulation and discuss in what practical circumstances each of the three option pricing methods (Parts I, II and III) is the most appropriate.

*Topics: Review of statistics and confidence intervals; Pricing path independent and dependent securities by simulation; Variance reduction techniques: antithetic and control variate techniques.*

### 6. Delta Hedging and Gamma Trading in Practice (2 lectures)

Black & Scholes meets the real world: We challenge the model's assumptions and partially investigate (via simulation) why data from observed market prices differ from theoretical values. We study the implications on delta and gamma trading from the perspective of an options market-maker. We explain how the "Greeks" are used to assess trading risk in practice.

*Topics: Static hedging versus dynamic hedging; Implied volatility; Delta and Gamma trading; Naked/Covered positions; Stop-Loss strategies; risk-management and "Greeks";*

### 7. Portfolio Optimization (6 lectures)

We begin with an overview of classical "single-period" Markowitz portfolio optimization and provide intuition on why asset diversification works in practice. We then introduce the relevant theoretical tools (linear Algebra, linear and non-linear programming) and practical tools (solver, Matlab engines, CVX) required for portfolio optimization. We study the standard mean-variance quadratic programming model, which we solve, both in closed form and numerically, for an arbitrary number of assets. We then study variations of the model based on alternative definitions of risk. We develop several data-driven applications and extensions in Excel/VBA and Matlab, on a variety of topics, including portfolio optimization with transaction costs, ETF indexation and pension fund surplus management.

*Topics: Introduction to optimization tools (solver, CVX); ADR and variance measures of risk; Linear and quadratic programming methods for portfolio management and asset allocation; The indexation problem and multiple linear regression; Surplus optimization; Mean-variance analysis with transaction costs; Testing and implementation issues; large-scale portfolio optimization.*

### 8. Multiperiod Portfolio Analysis (2 lectures)

We address the multi-period investment problem and the problem of choosing among efficient portfolios.

*Topics: Multiperiod portfolio models; Kelly's criterion; log-utility maximization; Portfolio insurance strategies.*

### 9. Bond Analytics (2 lectures)

We give a brief background on bond mathematics. We study the pricing of U.S. Treasury bonds and introduce some of the taxonomy used for fixed-income securities such as yield and duration.

*Topics: Review of discounting, present value and yield; Duration and convexity measures; Immunization and hedging applications; The discount factor and the spot yield curve.*

#### **10. Interest Rate Derivatives and Credit Models (2 lectures)**

We look at pricing of bonds and callable bonds, swaps and swaptions, caps and floors, and mortgage-backed securities. We give an introduction to the pricing of credit-sensitive securities and show how default models can be layered on top of the interest rate models. We do a case study of a convertible bond issue, in which we merge both interest-rate and credit aspects of pricing.

*Topics: Interest rate tree and the yield curve; Interest rate models: Ho-Lee, Black-Derman-Toy, Hull-White, and Heath-Jarrow-Morton; Callable bond pricing; Interest rate swaps and swaption pricing; Defaultable bond pricing;*

#### **11. High-frequency Trading and Market Microstructure (tentative 1-2 lectures)**

We discuss high-frequency trading, price impact and cutting edge algorithmic trading models, that are utilized by electronic trading desks and high-frequency trading firms. We give an overview of the latest academic research on this front. We also investigate the use of statistical arbitrage methods and illustrate how they are used in practice.

*Topics: Optimal control of a gains process; order book models; optimization of execution costs; statistical arbitrage; market microstructure.*

## Optional References

### “Industry” Books

The following books are widely read on Wall Street. Some books are edited compilations of research reports from the major investment banks.

- Against the Gods: The Remarkable Story of Risk, Peter Bernstein, Wiley, New York, 1998.
- Asset & Liability Management: A Synthesis New Methodologies, Risk Publications/Kamakura, London, 1998.
- Black-Scholes and Beyond: Option Pricing Models, Chriss Neil, Irwin Professional Publishing, Burr Ridge, Illinois, 1997.
- Bond Markets, Analysis and Strategies, 6rd edition, Fabozzi and Fabozzi, Prentice Hall, Englewood Cliffs, NJ, 2006.
- Capital Ideas: the Improbable Origins of Modern Wall Street, Peter Bernstein, Wiley, 2<sup>nd</sup> edition, New York, 2005.
- Corporate Hedging in Theory and Practice: Lessons from Metallgesellschaft, Eds. C. Culp and M.H. Miller, Risk Publications, 1999.
- Credit Derivatives: Application for Risk Management, Investment, and Portfolio Optimisation, Risk Publications, London, 1998.
- Derivative Credit Risk, RISK Publications/Renaissance Software, London, 1995.
- Derivatives Pricing: The Classic Collection, Peter Carr (Editor), Risk Books, 2004.
- Derivatives Trading and Option Pricing, Nicholas Dunbar (Editor), Risk Books, 2005.
- Efficient Asset Management: A Practical Guide to Stock Portfolio Optimization and Asset Allocation, Richard O. Michaud, HBS Press, 1998.
- Flash Boys: A Wall Street Revolt, Michael Lewis, W. W. Norton & Company, 2014
- Financial Futures Markets: Structure, Pricing and Practice, John Merrick, Jr., Harper and Row, New York, 1990.
- Fixed Income Analytics: State-of-the-Art Debt Analysis and Valuation Modeling, Ravi Dattatreya, ed., Probus Publishing, Chicago, 1991.
- Fixed-Income Portfolio Strategies, Frank Fabozzi, ed., Probus Publishing Company, Chicago, 1989.
- Fooled by Randomness: the Hidden Role of Chance in Life and in the Markets, Nassim Nicholas Taleb, Random House, 2nd edition, 2005.
- From Black-Scholes to Black Holes, RISK Publications/FINEX, London, 1992.
- Handbook of Mortgage Backed Securities, 5th edition, F. Fabozzi McGraw-Hill, 2001
- Hedging with Trees: Advances in Pricing and Risk Management Derivatives, Eds. M.Broadie and P. Glasserman, Risk Publications, London, 1998.
- Inventing Money: The Story of Long-Term Capital Management and the Legends behind It, Nicholas Dunbar, John Wiley & Sons, 2001
- Liar’s Poker; Michael Lewis, Norton, New York 1989.
- Monte Carlo: Methodologies and Applications for Pricing and Risk Management, Ed. Bruno Dupire, Risk Publications, London, 1998.
- My Life as a Quant: Reflections on Physics and Finance, Emanuel Derman, Wiley, 2004 Over the Rainbow, RISK Publications/Fuji, London, 1996,
- Rubinstien on Derivatives, Mark Rubinstein, Risk Publications, London, 1999.
- Stocks for the Long Run: the Definitive Guide to Financial Market Returns and Long-Term Investment Strategies, Jeremy Siegel, McGraw-Hill; 3rd edition, 2002.
- The Big Short: Inside the Doomsday Machine, Michael Lewis, Norton & Company, 2010
- The Black Swan: The Impact of the Highly Improbable, Nassim Nicholas Taleb, Random House, 2007
- The Complete Guide to Option Pricing Formulas, Haug Espen Gaader, McGraw-Hill, 2<sup>nd</sup> edition, 2006.
- The Concepts and Practice of Mathematical Finance, Mark S. Joshi, Cambridge University Press, 2003
- Fortune's Formula: The Untold Story of the Scientific Betting System that Beat the Casinos and Wall Street, William Poundstone, Hill and Wang, 2006
- The Handbook of Exotic Options: Instruments, Analysis, and Applications, ed.: Israel Nelken, Irwin, Chicago, 1995.
- The Handbook of Fixed Income Options, Frank Fabozzi, ed., Irwin Professional Publishing, Burr Ridge, Illinois, 1996.
- The Handbook of Fixed Income Securities, 5th edition, Frank Fabozzi, ed., Irwin Professional Publishing, Burr Ridge, Illinois, 1997.
- The Winner's Curse: Paradoxes and Anomalies of Economic Life, Richard Thaler, Princeton University Press, Princeton, NJ, 1994.
- Trading and Investing in Bond Options, M. Anthony Wong, John Wiley, New York, 1991.
- Volatility and Correlation: the Perfect Hedger and the Fox, Riccardo Rebonato, John Wiley & Sons; 2nd edition, 2004.
- Volatility: New Estimation Techniques for Pricing Derivatives, ed. Robert Jarrow, Risk Publications, London, 1998.
- When Genius Failed: The Rise and Fall of Long-Term Capital Management, Roger Lowenstein, Random House, 2000.

## **Journals**

Many articles relevant to this course appear in finance journals. Some of the journals that are pitched more towards practitioners include: Journal of Derivatives, Journal of Fixed Income, Journal of Portfolio Management, Journal of Computational Finance Financial Analysts Journal, and Risk Magazine. Some of the academic journals include: Journal of Finance, Journal of Financial and Quantitative Analysis, Journal of Futures Markets, and Review of Financial Studies.

## **Finance Textbooks**

Cox and Rubinstein, Options Markets, Prentice Hall, Englewood Cliffs, NJ, 1985.  
Dixit and Pindyck, Investment Under Uncertainty, Princeton University Press, Princeton, NJ, 1994.  
Elton and Gruber, Modern Portfolio Theory and Investment Analysis, 5th edition, Wiley, New York, 1995.  
Garbade Kenneth, Fixed Income Analytics, MIT Press, Cambridge, MA, 1997.  
Hull John, Options, Futures, and other Derivatives, 7th edition, Prentice Hall, NJ, 2008.  
Jarrow and Turnbull, Derivative Securities, 2nd edition, South-Western College Pub., 1999.  
Luenberger David, Investment Science, Oxford University Press, 1998.  
Markowitz, Mean-Variance Analysis in Portfolio Choice and Capital Markers, Blackwell, New York. 1987.  
Rebonato Riccardo, Interest-Rate Option Models: Understanding, Analysing and Using Models for Exotic Interest-Rate Options, 2nd edition, Wiley, NY, 1998.  
Rodriguez and Carter, International Financial Management, 3rd edition, Prentice Hall, Englewood Cliffs, NJ, 1984.  
Shreve Steven, Stochastic Calculus Models for Finance: Continuous Time Models, Springer 2005.  
Sundaresan, Suresh, Fixed Income Markets and their Derivatives, 2nd edition, South-Western, Cincinnati (2002).  
Tuckman, Bruce, Fixed Income Securities: Tools for Today's Markets, 2nd edition, Wiley, NY, 2002.  
Wilmott Paul, Paul Wilmott on Quantitative Finance, 2nd edition, John Wiley & Sons, 2006

## **Statistics Textbooks**

Newbold Paul, Statistics for Business and Economics, Prentice Hall, 1989.  
Smith Gary, Statistical Reasoning, Allyn and Bacon, Boston, 1985.

## **Time Series and Multivariate Statistics Textbooks**

Johnson and Wichern, Applied Multivariate Statistical Analysis, Prentice Hall, Englewood Cliffs, NJ, 1982.  
Maxwell, Multivariate Analysis in Behavioural Research, Wiley, New York, 1977.  
Pankratz, Forecasting with Univariate Box-Jenkins: Concepts and Cases, Wiley, New York, 1983.  
Pindyck and Rubinfeld, Econometric Models and Economic Forecasts, McGraw Hill, New York, 1976.  
Taylor, Modeling Financial Time Series, Wiley, New York, 1986.